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Trade, Schumpeterian Growth, and the Incentives for Innovation

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Abstract

The present study develops quality-ladder models of endogenous growth with and without firm heterogeneity. It analyses the effects of trade and trade liberalisation on the level of manufacturing productivity, on firm selection and on the investment into research. In this framework, I demonstrate that trade liberalisation unambiguously tightens firm selection and makes the economy more productive. Additionally, there is a reallocation of resources from non-exporting to exporting industries. Trade liberalisation increases the investment into research for sufficiently small, yet plausible, values of the elasticity of substitution.

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1 Introduction

The pioneering studies of Melitz (2003) and Bernard et al. (2003) introduced firm heterogeneity into the theoretical trade literature, in order to reconcile theory with the empirical evidence. This evidence suggests that there is considerable heterogeneity in firm size, profits and the export status.[1] The theoretical papers are both able to show that trade liberalisation, meaning a reduction in the costs of trade, push low-productivity firms out of the market, while increasing the market shares of high-productivity firms. As a result, average productivity rises in an economy. These changes are entirely static in nature, as there is no engine of growth incorporated in these models. Therefore, only the level of productivity rises, but not its growth rate. Although these early papers already go a long way in confirming the empirical results on firm heterogeneity, they cannot account for the effect of trade liberalisation on productivity growth. A key study linking trade liberalisation and economic growth was Sachs and Warner (1995), who showed that open economies outperformed closed economies on various measures of economic performance, including growth. A similar analysis, with more recent data, was conducted by Wacziarg and Welch (2008). They confirm the result that trade liberalisation has a positive impact on economic growth.[2]

Several studies subsequently introduced an engine of growth into the Melitz model framework. They can all confirm the result of Melitz (2003), namely that trade liberalisation increases the level of productivity. Baldwin and Robert-Nicoud (2008) consider various specifications of innovation technologies. Some of these specifications result in a positive link between trade liberalisation and growth, while others lead to a negative relationship. Unel (2010) conducts a similar study, with the crucial difference that the amount of technology diffusion increases with the level trade. Nevertheless, he also finds that trade liberalisation has an ambiguous effect on growth. Finally, Gustafsson and Segerstrom (2010) construct a semi-endogenous growth model with firm heterogeneity.[3] Their result depends on the size of spillovers from the total stock of knowledge accumulated. For weak spillovers, trade liberalisation boosts productivity growth in the short-run. However, when the spillovers are stronger, the result turn upside down and trade liberalisation hurts growth. In the long-run, productivity growth in completely independent of trade liberalisation. This result is due

[1] See for example Bernard and Jensen (1999) and Tybout (2001).

[2] Also see Alesina et al. (2000) or Rodrik and Rodriguez (2000) for further contributions on this issue.

[3] Semi-endogenous growth models are able to remove the scale effect of population, which bothers early models of R&D growth, because there is no positive empirical relation between population size and growth. However, the semi-endogenous growth models in turn suffer from the fact that population growth is required to achieve economic growth in the long-run.

to their semi-endogenous growth framework, where economic growth is entirely driven by population growth in the long-run. All these studies use the variety-expansion approach to model R&D. It is the dominant approach in the literature on trade and firm heterogeneity.[4] In contrast to these studies, Haruyama and Zhao (2017) employ a quality-ladder model of growth. They can demonstrate that trade liberalisation reallocates resources to R&D and unambiguously increases technological progress indefinitely. Thus, they manage to reproduce the empirical evidence presented above in a theoretical framework. However, they not only change the nature of the engine of growth, they also change the production function from CES (constant elasticity of substitution) to Cobb-Douglas, where the elasticity of substitution takes the value of 1. Thus, it is unclear whether the change in the result is due to the change in R&D technology, or due to the assumption of a Cobb-Douglas production function. Furthermore, the assumption of an elasticity of substitution equal to 1 is quite restrictive and implausible when looking at empirical evidence, which suggests much higher values.

The first objective of this study is therefore to fill this gap between Haruyama and Zhao (2017) on the one side and the studies of Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010) and Unel (2010) on the other side. I will construct a growth model of the quality-ladder type (like in Haruyama & Zhao) with a CES production (like in the other papers).

The second objective is to confirm the results of Haruyama and Zhao (2017) in this more general framework with respect to the elasticity of substitution. Although they managed to construct a model which can reproduce the empirical evidence on trade liberalisation and productivity growth, it is unclear whether this result is only achieved by the restrictive assumption of a Cobb-Douglas production function.

Finally, the last objective is to analyse the effects of introducing firm heterogeneity into a quality-ladder model of growth. More specifically, I want to find out whether the results on trade liberalisation and growth can also be found in the much simpler framework of firm homogeneity. This simple framework can of course not account for the empirical evidence on firm heterogeneity, which gave rise to the Melitz model in the first place. The objective is merely to find out whether trade liberalisation can increase growth in such a framework.

This study is structured as follows: Section 2 presents the model of Schumpeterian Growth with homogeneous firms. In section 3, firm heterogeneity and fixed costs are added. Finally,

[4] See for example Helpman et al. (2008) for an empirical paper employing this approach to estimate bilateral trade flows, or Melitz and Ottaviano (2008) for another theoretical paper.

section 4 analysis the effects of trade liberalisation in the framework presented in the previous section. Section 5 concludes.

2 Standard Schumpeterian Model

2.1 Consumers

There is a representative consumer, who maximises his lifetime utility according to:

$$U = \int_0^{\infty} e^{-\rho t} \ \ln Y(t) \ dt \tag{1}$$

where $Y(t)$ denotes the amount of final goods consumed in every period. Dynamic utility maximisation delivers the well-known Euler equation of consumption expenditure growth over time:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho \tag{2}$$

where $r(t)$ is the interest rate and $E(t)$ the expenditure on final goods by the consumer. The expenditure on final goods will grow over time if and only if the interest rate is larger than the discount rate ρ.

There are L number of workers in this economy and it is assumed to be constant over time. Labour is required in the manufacturing sector and in the R&D sector for innovation.

2.2 Production

2.2.1 Final Goods Production

The final output is produced under perfect competition, by assembling a range of intermediate products y. The production function takes the CES form:

$$Y(t) = \left\{ \int_0^1 \left[\sum_n q(n,i,t)^{\frac{1}{\sigma-1}} \cdot y(n,i,t) \right]^{\frac{\sigma-1}{\sigma}} di \right\}^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $q(n,i,t)$ denotes the quality vintage n in industry i at time t. Specifically, quality is given by $q(n,i,t) = \lambda^{n(i,t)} \cdot q(0,i,t)$. A quality improvement from any quality vintage j to

$j+1$ increases the quality level by λ. The quantity of intermediate goods is given by $y(n,i,t)$ and the elasticity of substitution by $\sigma > 1$, which implies gross substitutability across industries. Products in the same industry, but of different quality vintage, are perfect substitutes.[5]

2.2.2 Demand across different Quality Vintages

The final good producer has to solve two static optimisation problems. First, for every product i, the producer of the final good has to decide on the quality vintage he wants to use. Naturally, a higher quality vintage delivers more utility. However, they are also more expensive. The producer faces the intra-industry static optimisation problem:

$$\max_{y(\cdot)} \quad \sum_n q(n,i,t)^{\frac{1}{\sigma-1}} \cdot y(n,i,t)$$

subject to:

$$E(i,t) = \sum_n p(n,i,t)\ y(n,i,t),$$

where $E(i,t)$ denotes the expenditure in industry i at time t. The final good producer is indifferent between quality vintage n and quality vintage $n-1$ if:

$$\frac{p(n,i,t)}{p(n-1,i,t)} = \lambda^{\frac{1}{\sigma-1}} \tag{4}$$

where λ is the quality jump from vintage $n-1$ to vintage n. If the buyer is indifferent between two quality vintages, it is assumed that only the higher quality vintage is being purchased. This assumption implies that only the highest quality vintages is sold and consumed in equilibrium. Therefore, the summation sign drops from the production function.

2.2.3 Demand across different Products

After having established the demand for different quality vintages of the same product, the final good producer also has to allocate expenditure across the different products. The

[5]The specification of (3) has been used, for example, by Dinopoulos and Thompson (1998), Li (2001), Minniti et al. (2013) and Thompson and Waldo (1994).

inter-industry static-optimisation problem is

$$\max_{y(\cdot)} \quad \pi = P(t)Y(t)L - \int_0^1 p(n,i,t) \cdot y(n,i,t)\ di$$

subject to:

$$Y(t) = \left[\int_0^1 q(n,i,t)^{\frac{1}{\sigma}} \cdot y(n,i,t)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where $P(t)Y(t) = E(t)$ is the per capita expenditure on the final consumption good. $P(t)$ is the price of the final consumption good and also the relevant, quality-adjusted price index of intermediate products. The solution to this maximisation problem delivers the demand for intermediate products

$$y(n,i,t) = \frac{q(n,i,t)E(t)L}{P(t)^{1-\sigma}\ p(n,i,t)^{\sigma}} \tag{5}$$

The price index is given by[6]:

$$P(t) = \left[\int_0^1 q(n,i,t) \cdot p(n,i,t)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \tag{6}$$

According to (5), the demand for an intermediate product rises in its quality level $q(n,i,t)$ and decreases in its price $p(n,i,t)$.

2.2.4 Intermediate Goods Production

Intermediate goods are produced using labour only and there are constant returns to scale. The wage rate is normalised to one and the labour market is perfectly competitive. Firms face constant marginal costs of production c. There are no fixed costs of production.

The pricing decision of the industry leader (the firm with the highest quality level for a given industry) depends on the size of the innovation λ. In quality-ladder models, there is price competition from below for every monopolist. According to (4), the maximal price the industry leader can charge depends on the marginal costs of the previous industry leader, in order to drive him out of the market (due to the assumption that in case of indifference, only the higher quality good is being purchased). This constrained monopoly price is given

[6] The price index can be derived by plugging the demand for intermediate products (5) into the production function (3) and solving for P.

by

$$p = \lambda^{\frac{1}{\sigma-1}} c$$

Ideally, the monopolist wants to charge an unconstrained monopoly price to maximise his profits. As with all CES functions, the unconstrained monopoly price is a constant markup over marginal costs. Specifically, we have

$$p = \frac{\sigma}{\sigma - 1} c$$

Which price will be charged depends on the size of the innovation. In case of drastic innovation the unconstrained monopoly price will be charge. In case of non-drastic (incremental) innovation, the constrained monopoly price is used. The pricing decision can be summarised by:

$$p = c \cdot \theta, \quad \text{where} \quad \theta \equiv \begin{cases} \lambda^{\frac{1}{\sigma-1}}, & \text{for } \lambda < (\frac{\sigma}{\sigma-1})^{\sigma-1} \\ \frac{\sigma}{\sigma-1}, & \text{for } \lambda \geq (\frac{\sigma}{\sigma-1})^{\sigma-1} \end{cases} \tag{7}$$

Moving forward, assume that $\lambda \geq (\frac{\sigma}{\sigma-1})^{\sigma-1}$. That is, firms will charge the unconstrained monopoly price. Therefore, the profits will be:

$$\pi(t) = \frac{\sigma^{-\sigma}(\sigma - 1)^{\sigma-1} q E(t) L}{P(t)^{1-\sigma}} c^{1-\sigma} \tag{8}$$

2.3 R&D Investment

Firms engage in a R&D race to discover a new quality vintage of an existing product. It is assumed that every firm faces the same R&D technology, be it a potential entrant or an already existing firm. By Arrow's Replacement Effect, no incumbent will ever invest in R&D, because he would only destroy his current monopoly position without generating much additional profits from an innovation. The innovation technology is given by:

$$I(t) = \frac{R(t)}{f_r}, \tag{9}$$

where $R(t)$ is the number of researchers employed and f_r is a cost parameter. Let $V(t)$ denote the present value of profits π. Any firm willing to engage in an R&D races must choose the optimal number of researchers. If it is successful, it gains the flow of profits until it is being replaced by some new entrant. Therefore, the firm has to solve

$$\max_{R(t)} \quad \left\{ V(t)\frac{R(t)}{f_r} - R(t) \right\} \tag{10}$$

The first order condition of this optimisation problem is:

$$V(t) = f_r \qquad \text{for } R(t) > 0 \tag{11}$$

Because there is free entry into research, there are no profits left when accounting for the research costs as well. Otherwise, there would be unlimited entry in order to exploit the profits.

2.4 The Stock Market

Through the stock market, consumers channel their savings to firms in order to finance the up-front costs of R&D. The valuation of every firm is the present value of profits it can generate if it successfully innovates. In every time period, the shareholders receive a dividend payment equal to the profits π generated in that period. Additionally, there may be a valuation effect $\dot{V}(t)$. That is, the value of the firm may appreciate or depreciate. If a new quality vintage is invented in this industry by some other firm, then the shareholders lose the entire value of the stock. This event occurs at probability $I(t)$. The more investment into R&D is being made, the more probable a replacement becomes. Since investors should be indifferent between investing into a stock and receiving a risky, yet higher return or receiving the risk-free rate of return $r(t)$, the no-arbitrage condition for the stock market is given by:

$$r(t)V(t) = \pi(t) + \dot{V}(t) - I(t)V(t)$$

In equilibrium, the value of the firms will be constant. That is, we have $\dot{V}(t) = 0$.[7] Therefore, we have that:

$$V(t) = \frac{\pi(t)}{r(t) + I(t)} \tag{12}$$

[7]Differentiate (11) with respect to time to prove that $\dot{V(t)} = 0$. Under some more complex innovation technologies, this property need not be true. See for example Minniti et al. (2013) for an innovation technology, which implies changes in firm valuation over time.

According to this equation, the profits will be discounted by the risk-free interest rate $r(t)$ and the probability of being replacement by some new firm, which is given by $I(t)$. The larger the investments into R&D are, the more discounting occurs, therefore the lower the value of the firms will be.

2.5 The Labour Market

Labour is required for two purposes, for research and for manufacturing. Since the wage rate is normalised to one, total manufacturing labour demand equals its expenditure. Therefore, manufacturing labour demand is given by:

$$l_m = \int_0^1 y(i,t)c \quad di, \tag{13}$$

where $y(i,t)$ is given by (5). In order to solve this equation, we require the price index, which is given by (6). Substituting in the optimal pricing decision of firms, the price index can be expressed as:

$$P(t)^{1-\sigma} = Q(t)\left(\frac{\sigma}{\sigma-1}c\right)^{1-\sigma} \tag{14}$$

where $Q(t) = \int_0^1 q(it)di$ denotes average quality across all industries. Quite clearly, the price index is falling in average quality. Using this equation, manufacturing labour demand l_m can be calculated:

$$l_m = \frac{\sigma-1}{\sigma}E(t)L \tag{15}$$

According to this equation, manufacturing labour demand is a constant fraction of total expenditure $E(t)L$. Then, labour demand equals:

$$L = R(t) + l_m \tag{16}$$

where l_m is given by (15).

2.6 Autarky Equilibrium

Using the Euler equation (2), the equation for profits (8), the innovation technology (9), optimal R&D investment (11), present value of profits (12) and the price index (14), we can derive a condition for R&D investment in equilibrium:

$$\frac{\sigma^{-1}E(t)L}{\rho+\frac{R(t)}{f_r}}=f_r \tag{17}$$

where the assumption of symmetry is made. Specifically, I assume that $q(i) = Q$, the firm analysed here has average quality. In equilibrium, we have that consumption expenditure is constant, i.e. $\dot{E}(t) = 0$. This implies, from the Euler equation, that $r(t) = \rho$, the risk-free interest rate equals the discount rate and is thus constant. The left-hand side is the value of innovation and the right-hand side the sunk costs for developing a new quality vintage $n+1$. Equations (16) and (17) constitute a system of two equations in two unknowns, $E(t)$ and $R(t)$. Solving (17) for $E(t)$ and plugging it into (16), we get the number of workers employed in research:

$$R^A=\frac{1}{\sigma}\Big(L-(\sigma-1)f_r\rho\Big) \tag{18}$$

The superscript A denotes "autarky". The number of researchers employed is decreasing in the elasticity of substitution σ. This result is quite intuitive, as a larger σ increases the competition among the industries. This compromises a firms pricing power and therefore reduces the markup it can charge. A lower markup implies lower profits, which in turn reduces the attractiveness of investment into R&D in the first place. R is also decreasing in the cost parameter f_r. A higher f_r requires a larger value of present discounted profits. This can only be achieved if the number of researchers decreases, thereby decreasing the rate of replacement. A higher time discount rate ρ also reduces the number of researchers, as people now care less about the future and the future gains from innovation, therefore investing less into research.

From the number of researchers, the rate of technological progress can be calculated. In this model, technological progress occurs due to quality improvements of a fixed variety of products. We have defined average product quality to be

$$Q(t)=\int_0^1 q(i,t)\quad di$$

A quality jump from any quality vintage n to vintage $n+1$ increases the quality by the factor λ. Innovation occurs with probability $I(t)$. Differentiating $Q(t)$ with respect to time delivers:

$$\dot{Q}(t) = (\lambda - 1)I(t)Q(t)$$

Making use of equations (9) and (18), the rate of technological progress is given by:

$$g^A \equiv \frac{\dot{Q}(t)}{Q(t)} = \frac{\lambda - 1}{\sigma f_r}\Big(L - (\sigma - 1)f_r\rho\Big) \tag{19}$$

As all first generation Schumpeterian growth models, there is a scale effect of population. Larger economies should grow faster than smaller ones.

Having established the number of researchers and correspondingly, the growth rate of technology, the only thing left is the final goods consumption expenditure $E(t)$.

$$E^A = 1 + \rho\frac{f_r}{L} \tag{20}$$

Expenditure in fact equals the intertemporal budget constraint of the consumer in steady state. Given that the wage is normalised to one, the first term on the right-hand side denotes labour income. The second term can be interpreted as interest income from equity investment. Remember that firms require funding to finance their upfront R&D investments of f_r. This funding is channeled through the stock market, as described above. Given that the measure of industries is one, the total investment in the economy equals f_r. Every consumer owns a share $1/L$ of the total amount and it bears an interest rate of ρ. Therefore, $\rho\frac{f_r}{L}$ is the interest income on equity investment for every consumer.

2.7 Trade Equilibrium

So far, we did not allow for any trade to happen. Now, let there by two countries, name them domestic and foreign. They both function in the way described above and are structurally identical. Assume they open up to trade with one another, allowing intermediate goods to be transported from one country into the other at trade costs of $\tau > 1$. These trade costs are modelled as iceberg trade costs, meaning that a unit τ must be shipped in order for one unit to arrive at the destination. Therefore, the effective marginal cost of production, in case of exports, is now $\tau \cdot c$. It is assumed that there is diffusion of technology, meaning that domestic (foreign) firms can also use the knowledge created by foreign (domestic) firms for their own innovations.

The demand for final and intermediate goods, the optimal pricing strategy and the innovation technology are all taken over from the previous subsection. What changes are the profits (and correspondingly the stock market), the labour market and the price index. Start with the profits. Every firm that successfully invents a new quality vintage will sell its product both at home and abroad. Profits from the domestic market are given by (8). Abroad, firms charge the price:

$$p_x = \frac{\sigma}{\sigma - 1} c\tau \tag{21}$$

The subscript x stands for export. Correspondingly, profits earned in the foreign market are:

$$\pi_x(t) = \frac{\sigma^{-\sigma}(\sigma - 1)^{\sigma-1} qE(t)L}{P(t)^{1-\sigma}} (c\tau)^{1-\sigma} \tag{22}$$

Clearly, the profits are decreasing in the trade costs τ. This is because higher trade costs also increase the price charged (due to the CES production function). However, the increase in the price reduces the demand for the product and thereby also the profits.

Concerning the stock market, there is a change in the replacement rate, which was previously given by $I(t)$. Now, given that there are two economies and the assumption of knowledge diffusion, the new replacement rate is $2I(t)$. The probability of being replaced is now twice as large as before. The no-arbitrage condition of the stock market is therefore given by

$$rV_x(t) = \pi(t) + \pi_x(t) + \dot{V}_x(t) - 2I(t)V_x(t)$$

In equilibrium, we will again have that $\dot{V}(t) = 0$ and consequently, that the present discounted value of profits is

$$V_x = \frac{\pi + \pi_x}{\rho + 2I} \tag{23}$$

Labour for manufacturing is now only needed in one half of all industries. However, in these industries, there is labour demand such that both markets are satisfied. Manufacturing labour demand is thus:

$$l_m = \int_0^{1/2} y(i)c \quad di + \int_0^{1/2} y_x(i)c\tau \quad di$$

$$= \frac{1}{2}(1+\tau^{1-\sigma})\frac{Q(t)E(t)L}{P(t)^{1-\sigma}}c^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}$$

where $y_x(i)$ indicates that the export price (21) is used for the intermediate demand from abroad. Intermediate demand is still given by (5).

Again, we require the price index to solve for the manufacturing labour demand. Half of the intermediate products used for the production of the final good are produced at home, the other half are produced abroad. The imported products are more expensive, as the exporter fully charges the trade costs on the consumers. This implies that the price index under trade is larger than under autarky, where we had no trade costs. Knowing this, the price index is given by:

$$P(t)^{1-\sigma} = \frac{1}{2}(1+\tau^{1-\sigma})Q(t)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}c^{1-\sigma} \tag{24}$$

For the case of $\tau = 1$, the price index simplifies to the autarky price index (14). Plugging the price index into the above equation, we get the manufacturing labour demand

$$l_m = \frac{\sigma-1}{\sigma}E(t)L \tag{25}$$

Note that this equation is identical to the one in the autarky equilibrium (15). The trade costs have no impact on the manufacturing labour demand. The intuition is the following: higher trade costs imply that exporters sell less on the foreign market, because they have to charge a higher price than before in order to keep the markup fixed. From this, it immediately follows that the local producers sell more of their products, to compensate for the reduction in imported goods. However, every firm in the market serves both the domestic and the foreign market. So, an increase in the trade costs reduces the exports, but at the same time increases the number of products sold at home, because the foreign products became more expensive. Due to the assumption that the two countries are identical, the two effects cancel out and the labour demand is independent of the trade costs.

Using the Euler equation (2), the equations for profits (8) & (22), the innovation technology (9), the present value of profits (23) and the price index (24), we can again derive a condition for R&D investment in equilibrium:

$$\frac{2\sigma^{-1}E(t)L}{\rho + 2\frac{R(t)}{f_r}} = f_r \tag{26}$$

where we again analyse the symmetric case of $q(t) = Q(t)$, meaning the industry has an average quality level. Already from this equation it becomes apparent that the number of researchers under trade must be larger than under autarky. Any firm willing to invest into R&D has to invest sunk costs of f_r. This holds both under autarky and under trade. However, under trade, the profits a firm can generate if it successfully invents a new quality vintage are double the profits under autarky, because it can serve both markets. At the same time, trade implies that the competition of potential entrants has doubled as well, which is represented by $2\frac{R(t)}{f_r}$, the replacement rate. So, we have that the numerator in the left-hand side has doubled, while the denominator has not quite doubled. This, however, cannot be an equilibrium, as the left-hand side should equal the sunk costs of innovation, f_r. Therefore, the number of researchers has to increase such that the incentive condition for innovation is fulfilled.

Equations (16) and (26) form a system of two equations in the two unknowns $R(t)$ and $E(t)$. Applying the same procedure as above, we can derive the number of researchers in a trade regime:

$$R^T = \frac{1}{\sigma}\left(L - \frac{\sigma - 1}{2} f_r \rho\right) \tag{27}$$

The superscript T denotes "trade". This equation is very similar to the one derived in the autarky case (18). The only difference is the term $\left(\frac{\sigma-1}{2}\right)$, which was previously $(\sigma - 1)$. Therefore, we have that R^T is strictly greater than R^A, the number of researchers under trade is larger than under autarky. The opening of the economy creates stronger incentives for research and innovation. Firms can now spread their fixed costs f_r over twice the number of consumers, making it much easier to recover them.

From the number of researchers, we can again calculate the rate of technological progress, which directly depends on the number of researchers.

$$g^T = \frac{2(\lambda - 1)}{\sigma f_r}\left(L - \frac{\sigma - 1}{2} f_r \rho\right) \tag{28}$$

Obviously, the growth rate of productivity under trade g^T is also larger than under autarky g^A, as the number of searchers is larger under trade. Additionally, there are now two countries investing into research, not just one, which further boosts the growth rate. We can therefore conclude that:

Proposition 1. *Moving out of autarky increases the rate of technological progress.*

Finally, consumption expenditure on final goods is given by:

$$E^T = 1 + \rho \frac{f_r}{2L} \tag{29}$$

The interpretation is similar under autarky. The first term on the right-hand side labour income and the second term interest income from equity investment. Interest income is now smaller than under autarky. This is because the same number of firms, therefore the same amount of equity investments, exits under trade as under autarky. However, under trade, foreigner can also investment in domestic firms, which implies that every individual only owns a share of $1/2L$ instead of $1/L$ as before. Therefore, individuals now hold less equities than under autarky. Naturally, this also leads to smaller interest payments from these equities[8]. The level of consumption of final goods has therefore decreased.

2.8 Welfare

The representative agent derives utility from consumption of the final good, $Y = E/P$. Since E is constant in equilibrium, the only thing determining the growth rate of consumption is the quality index $Q(t)$, which increases over time. The increase in quality reduces the price index, thereby increasing consumption at the same rate. Using the equations for final consumption expenditure (20) & (29) and the price indices (14) & (24), the intertemporal

[8]This results follows directly from the fact that the number of firms is fixed at the measure 1. In an expanding-variety model, where trade leads to a larger number of firms, this need not be the case.

utility function can be re-expressed as:

$$\rho U^A = \overbrace{\ln\left(1+\rho\frac{f_r}{L}\right)}^{\text{Expenditure Effect}} - \overbrace{\ln\left(\frac{\sigma}{\sigma-1}c\right)}^{\text{Price Effect}} - \overbrace{\frac{1}{1-\sigma}\frac{g^A}{\rho}}^{\text{Growth Effect}} \tag{30}$$

$$\rho U^T = \overbrace{\ln\left(1+\rho\frac{f_r}{2L}\right)}^{\text{Expenditure Effect}} - \overbrace{\left[\ln\left(\frac{\sigma}{\sigma-1}c\right) + \frac{1}{1-\sigma}\ln\left(\frac{1+\tau^{1-\sigma}}{2}\right)\right]}^{\text{Price Effect}} - \overbrace{\frac{1}{1-\sigma}\frac{g^T}{\rho}}^{\text{Growth Effect}} \tag{31}$$

Utility is determined by three effects: the expenditure effect, the price effect and the growth effect. The second and third effect jointly pick up the effect of the price index. Technically, the growth effect is also part of the price effect, as it represents the effect of a continuously falling price index due to continuous quality improvements. So, the growth effect captures the effect of technological progress on welfare.

As already discussed, consumption expenditure is smaller under trade than under autarky. This is due to the fact that consumers hold fewer assets, which in turn reduces the interest income from these assets, given that the interest rate is fixed at ρ in both cases. The price effect is larger under trade than under autarky. This is because half of all products are imported and therefore include trade costs τ. In case of $\tau = 0$, the price effects would be the same.[9] By proposition 1, growth under trade is larger than under autarky. So, we have two effects diminishing welfare when moving to a trade regime and one effect which increases welfare. However, the growth effect is scaled up by the time discount rate ρ. Therefore, it may well outperform the expenditure and the price effect such that trade is welfare-improving overall. Although a definitive answer cannot be given without assuming certain values for the parameters.

2.9 Trade Liberalisation

Actually, the more interesting, and empirically more relevant question, is whether trade *liberalisation* is growth- and welfare-improving, since virtually all countries are already engaged in trade. Here, trade liberalisation is modelled by a decrease in the trade costs τ. Clearly,

[9] This effect is due to the assumption of constant returns to scale. In case of increasing returns to scale, opening up the economy leads to lower marginal costs due to increased scale. Then, it may well be that prices under trade are lower than under autarky, even in the presence of trade costs.

the number of researchers R and the expenditure on final goods E are independent of the trade costs. This immediately implies that the rate of technological progress is also independent of the trade costs.[10] The only thing that depends on the trade costs is welfare. It does so through the price effect. Trade liberalisation reduces the price charged for intermediate products, thus monotonically increasing welfare of the consumers.

Trade in a standard Schumpeterian Growth Model is definitively research and growth promoting. However, trade liberalisation does not increase the incentives for research and innovation any further, in contrast to the empirical evidence presented earlier. Therefore, the next section introduces firm heterogeneity and fixed costs of production and of exports. Firm heterogeneity will be modelled in the way that firms receive a certain value of marginal costs, which differs from their competitors. Haruyama and Zhao (2017) have shown that in a model of Schumpeterian Growth with firm heterogeneity, trade liberalisation unambiguously increases the number of researchers and the growth rate of technological progress. However, they have assumed a Cobb-Douglas production function, i.e. an elasticity of substitution $\sigma = 1$. The next section follows Haruyama and Zhao (2017) in most aspects, except for the production function. It will be of a CES form, as in this section here.

3 Introducing Firm Heterogeneity

3.1 Intermediates

Many things can be taken over from the previous section and will therefore not be repeated again here. Discussed here are only these subsections in which changes are being made.
Firms differ in their marginal costs of production. The values are drawn, for now, from a unspecified distribution function

$$Z(c), \quad c \in (0, c_H), \quad 0 < c_H < \infty \tag{32}$$

where c_H is some upper bound of possible values. The true value of the marginal cost will only become known to the firm after it successfully invented a new quality vintage. That is, after it participated in the R&D race. Therefore, during the R&D race, there is uncertainty

[10] Gustafsson and Segerstrom (2010) arrived at the same result in their model, due to the usage of a semi-endogenous growth model.

about the profitability of a potential innovation, a feature which appears to be highly realistic. Previously, the profits from innovation were known to all firms, since the marginal costs were deterministic. Again, there are iceberg trade costs of τ.

The assumption of technology diffusion has to be slightly adjusted. Previously, all firms, be they domestic or foreign, could use the knowledge of the current quality leader in any industry for their own research in order to create a new blueprint of higher quality vintage. This still holds true. Additionally, there is also knowledge diffusion about the second-highest quality vintage in the way that the second-highest quality good can be competitively produced at marginal cost c_L by any firm in any country. Therefore, the quality leaders face homogeneous price competition from below.[11] This assumption implies that the maximal technological gap in every industry between the trading countries is 1.
The maximal price the quality leader can charge to drive competition out of the market is

$$p = \lambda^{\frac{1}{\sigma-1}} c_L$$

The optimal price firms want to charge is still the same as in the previous section, namely $p = \frac{\sigma}{\sigma-1}c$. Due to the heterogeneity in marginal costs, this optimal price differs across firms. Firms will want to set this unconstrained monopoly price. However, they can only do so if their marginal cost is sufficiently low. The condition is:

$$c < \frac{\sigma - 1}{\sigma} \lambda^{\frac{1}{\sigma-1}} c_L \tag{33}$$

Obviously, not all firms can fulfil this condition. In case a firm cannot do so, it can only charge the constrained monopoly price $\lambda^{\frac{1}{\sigma-1}} c_L$ to overcome the competition from the lower quality vintage in the same industry. In Haruyama and Zhao (2017), with the Cobb-Douglas production function, firms will always charge the constrained monopoly price, since they can never fulfil (33), as $\sigma = 1$ in their case. The optimal pricing strategy for firms can be summarised by:

$$p = \begin{cases} \lambda^{\frac{1}{\sigma-1}} c_L, & \text{for } c > \frac{\sigma-1}{\sigma}\lambda^{\frac{1}{\sigma-1}} c_L \\ \frac{\sigma}{\sigma-1} c, & \text{for } c \leq \frac{\sigma-1}{\sigma}\lambda^{\frac{1}{\sigma-1}} c_L \end{cases}$$

[11] Homogeneous in this case means that the price competition from below is the same for all industries. Without this assumption of technology diffusion, the price competition from below would be different in every industry, depending on the marginal costs of the previous quality leader.

The level of profits earned by a firm depends on its marginal costs. Firms with a high value of marginal costs have to charge the constraint monopoly price and earn domestic profits of

$$\pi(t,c) = \left(1 - \frac{c}{\lambda^{\frac{1}{\sigma-1}} c_L}\right) \frac{qE(t)Lc_L^{1-\sigma}}{P(t)^{1-\sigma}\lambda}, \tag{34}$$

where the intermediate demand function (5) is used. Clearly, non-negative profits can only be earned if the marginal costs are lower than the maximal price which can be charged. Any firm above this threshold will not enter production in the first place, as it cannot cover its fixed costs.

Firms operating in the middle range of marginal costs will charge the unconstrained monopoly price and earn domestic profits

$$\pi(t,c) = \frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}qE(t)L}{P(t)^{1-\sigma}} c^{1-\sigma} \tag{35}$$

In case of sufficiently low marginal costs, a firm can additionally also export its product and thus earn profits in the foreign market of

$$\pi_x(t,c) = \frac{\sigma^{-\sigma}(\sigma-1)^{\sigma-1}qE(t)L}{P(t)^{1-\sigma}} (c\tau)^{1-\sigma} \tag{36}$$

The profits from exporting are decreasing in the trade costs τ, through the demand effect. The trade costs increase the price charged, thereby reducing demand for the product and thus also the profits generated. Whichever profit function applies for a specific firm, the profits are decreasing in the realised value of marginal costs. In the case of (34), this is due to the decreasing profits margin. In the cases of (35) & (36), although the profit margin can be held constant (due to the CES pricing), profits subside nevertheless due to decreasing demand for the product.

3.2 Entry decision

Any firm that wants to enter production has to incur multiple types of fixed costs. First, the firm has to invest into research in order to develop a new quality vintage $n+1$. These **R&D costs** are sunk, a firm cannot recover these expenses, even if it decides not to enter the market. As in the previous section, these costs are denoted with f_r. At this point in time, the value of marginal costs is unknown to the firm conducting the research. Second, after successfully innovating, the marginal costs are revealed and the firm has to decide whether to enter the market and start producing or not to enter the market and abandon the innovation. If the firm decides to enter the market, it has to pay fixed costs f to start production. These **costs of production** can be interpreted as the costs of setting up the factory or the costs to fulfil the applicable regulatory requirements. Third, a firm must pay **export costs** f_x if it decides to export its products to the foreign market. All these costs are sunk costs, they have to be paid upfront. Different than in the previous section, entry into production and into export are now active choices by the firms. They can decide to enter or not to. The two choices are also separate. Not every firm in production will also decide to export (of course every exporter also produces domestically). This "selection into exporting" is ensured by the trade costs τ.[12]

Denote the present discounted values of profits from the domestic and foreign market by $v(c)$ and $v_x(c)$, respectively. For a firm to enter production, the present discounted value of profits must be sufficiently large to cover the fixed costs of entry f. That is, for $v(c) \geq f$, the firm will enter production. In case of $v(c) < f$, it will not enter production, the investments into research have been in vain and the incumbent remains in position. The knowledge created by the investment is lost, no other firm can use for it's own production or research. As noted before, profits monotonically decrease in the value of marginal costs. Therefore, there exists a unique value of marginal cost, call it C, where firms are indifferent between entering production and abstaining. Specifically, the cut-off condition is:

$$v(t, C) = f \tag{37}$$

Any firm with $c \leq C$ will enter production.

The decision to export is quite similar to the one concerning domestic production. Firms export their products if $v_x(t, c) \geq f_x$. That is, if the profits from exporting sufficiently large to cover the fixed costs of exporting. Again, there exists a unique cut-off value of marginal cost, where firms are indifferent between exporting and not exporting. The cut-off value is

[12]The presence of trade costs is sufficient even in the case of $f_x = f$, that is, if the fixed costs were the same. As discussed, the trade costs reduce profits. Therefore, in order to generate the same level of profits (to pay the same amount of fixed costs), a firm requires lower marginal costs if it also wants to export. So, there is selection into exporting.

defined by:

$$v_x(t, C_x) = f_x \tag{38}$$

Firms with marginal costs of $C_x \leq c \leq C$ serve the domestic market, but do not export. A visual representation of the entry decision of firms is depicted in figure 1.

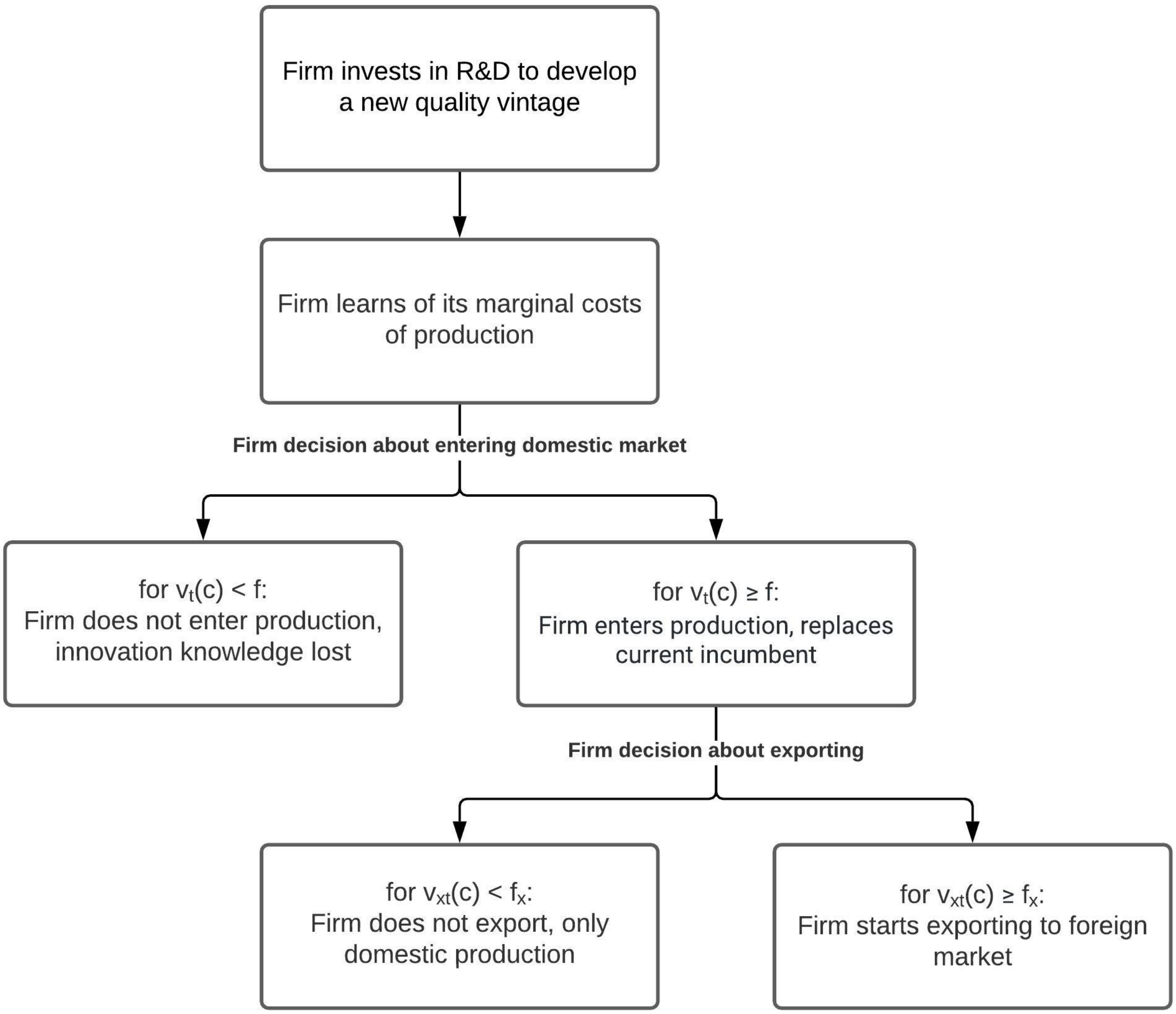

Figure 1: Entry decision of firms

3.3 Industry Dynamics

As in Haruyama and Zhao (2017), industries can be put into two categories. In the type-A industries, the highest quality vintage is traded, in the type-B industries, it is not traded. Expressing this in terms of marginals costs, an industry is of type A if the marginal cost of the quality leader is $c \leq C_x$. The measure of industries in this category is denoted with N_A.[13] If the marginal cost of the quality leader is $C_x \leq c \leq C$, then he does not export his product. Thus, the highest quality is only available to domestic consumers, while the foreign consumers have to settle for the second highest quality. Due to the assumption of technology diffusion, the second highest quality becomes available at competitive prices, irrespective of where it has been invented in the first place. Denote these industries by N_B. Then, we have that

$$1 = N_A(t) + N_B(t), \tag{39}$$

as the total number of industries is one. The subscript t indicates that these measures of industries are subject to change over time. However, not only do the measures change, also the composition within them changes. As innovation occurs in either country, the industry can change from being type-A to type-B, or vice versa, or remain of the same type. Thus, this model allows for reversals in trade flows, a feature which does not exist in expanding-variety models. The measure of type-A industries changes according to:

$$\dot{N}_A(t) = N_B(t)Z(C_x(t))(2I(t)) - N_A(t)[Z(C(t)) - Z(C_x(t)]\ (2I(t)) \tag{40}$$

The first term on the right-hand side denotes inflows into type-A industries and the second term denotes outflows. As established above, an industry is of type-A if the incumbent has marginal costs of $c \geq C_x$. Then, $Z(C_x)$ is the probability of receiving such a marginal cost and $Z(C_x)(2I(t))$ is the arrival rate of such an innovation over an infinitesimally small time period, given that $2I$ is the total investment into research in both countries in that time period. Similarly, the instantaneous probability of an innovation with marginal costs of $C_x \leq c \leq C$ is given by $[Z(C) - Z(C_x)](2I(t))$.

[13] N_A can also be interpreted as a measure of trade openness. The more open an economy is, the more exporting industries exist.

3.4 The Stock Market

Due to the presence of fixed costs of production, the value of firms and the value of innovation are no longer the same. Therefore, a slight change in notation becomes necessary. Let $v(c)$ denote the present discounted value of profits $\pi(c)$. The asset pricing equation for the domestic market is then given by

$$r(t)v(t,c) = \pi(t,c) + \dot{v}(t,c) - Z(C(t))\ (2I(t))\ v(t,c), \tag{41}$$

where $Z(C)(2I(t))$ is the instantaneous probability of losing all market value. This is either due to being replaced by the newcomer and thus losing all demand, or because the now second-highest quality good is produced at competitive prices, implying that no profits can be generated. In the foreign market, similarly, $v_x(c)$ is defined by:

$$r(t)v_x(t,c) = \pi(t,c) + \dot{v}(t,c) - Z(C(t))\ (2I(t))\ v_x(t,c) \tag{42}$$

Firms are valued according to the profits they generate. The present discounted values of profits are ex post values, that is, after the marginal costs has been revealed. However, firms have to make the R&D investment decision ex ante, before the marginal cost has been revealed. The uncertainty about future profits imply that firms have to work with expectations about profits. As in the previous section, firms will invest up to the point where the value of innovation equals the costs of innovation. Let $V(t)$ denote the ex ante value of innovation. The value of innovation is given by the value of the firm $v(c)$ minus the fixed costs of production:

$$V(t) = \int_0^{C(t)} [v(t,c) - f]\ dZ(c) + \int_0^{C_{xt}} [v_x(t,c) - f_x]\ dZ(c) \tag{43}$$

In equilibrium, we again have that:

$$V(t) = f_r$$

3.5 The Labour Market

Theoretically, the upper limit on the marginal cost is given by $C < \lambda^{\frac{1}{\sigma-1}} c_L$. Any value above it would lead to non-positive profits, which means that the fixed costs of entry could not be covered. However, I impose the assumption that $C \leq \frac{\sigma-1}{\sigma}\lambda^{\frac{1}{\sigma-1}} c_L$. That is, no firm which would charge the constraint monopoly price $\lambda^{\frac{1}{\sigma-1}} c_L$ ever enter the market in the first place, simply because it cannot generate enough profits to cover the fixed costs f. Therefore, only firms with CES pricing are in the market.[14]

Labour is required for four purposes: in research, in manufacturing and for the fixed costs of production and export. Over an infinitesimally small time interval, the number of successful (and implemented) innovations in one country is $Z(C)I$. Out of these innovations, exactly $Z(C_x)I$ are also exported. So, the total number of workers required for the fixed costs are $fZ(C)I$ and $f_x Z(C_x)I$, respectively.

The construction of the manufacturing labour demand is confined to appendix A. It can be summarised by:

$$l_m(t) = \frac{\sigma - 1}{\sigma} E(t)L \ \Theta(C, C_x; \tau, \sigma) \tag{44}$$

where

$$\Theta(C, C_x; \tau, \sigma) = \frac{\Xi(C, C_x; \tau, \sigma)}{\Omega(C, C_x; \tau, \sigma)}$$

3.6 Trade Equilibrium

Given that there are four unknowns in this model, namely the cut-off values C and C_x, the level of expenditure E and the number of researchers R, we need four equation to solve this model.

Using the asset pricing equations, we get the value of present discounted profits in the domestic and foreign market:

$$v(c) = \frac{\pi(t, c)}{r(t) + 2Z(C)I(t)} \qquad v_x(c) = \frac{\pi_x(t, c)}{r(t) + Z(C)I(t)} \tag{45}$$

[14] This assumption might seem to be quite restrictive. However, there are two reasons why I decided to impose it anyway. First, Haruyama and Zhao (2017) have already shown that in the Cobb-Douglas case, where only constraint pricing is used, trade liberalisation increases the number of researchers. Thus, with this restriction, we can analyse the unconstrained pricing option in isolation. Second, allowing for both pricing options would simply make the equations overwhelmingly complicated.

At the cut-off values C and C_x, these present discounted profits must exactly equal to the fixed costs of production and export, respectively. Thus, using the Euler equation (2), the innovation technology (9), the profits (35) & (36) and the price index (A.1), the conditions determining the cut-off values can be expressed as:

$$f = \frac{\sigma^{-1} q Q^{-1} \Omega^{-1} EL}{\rho + 2Z(C)R/f_r} C^{1-\sigma} \qquad f_x = \frac{\sigma^{-1} q Q^{-1} \Omega^{-1} EL}{\rho + 2Z(C)R/f_r} (C_x \tau)^{1-\sigma} \tag{46}$$

where Ω is the function from the price index. As the left-hand sided of the equations are constants, the right-hand sides must adjust such that the equations hold. For example, if the population L increases, then the cut-off values must adjust accordingly.

The cut-off levels are of course determined ex post. However, the research decision is done ex ante. The ex ante value of innovation must equal to the fixed costs of innovation f_r, given that there is free entry into research. Using equation (43) on the ex ante value of innovation and the present discounted values of profits, the following condition on the number of researchers can be established:

$$\tilde{F}(C, C_x) = \frac{\sigma^{-1} q Q^{-1} EL}{\rho + 2Z(C)R/f_r} \Lambda(C, C_x; \tau, \sigma) \tag{47}$$

where

$$\tilde{F}(C, C_x) = \frac{f_r}{Z(C)} + f + \frac{Z(C_x)}{Z(C)} f_x \tag{48}$$

and

$$\Lambda(C, C_x; \tau, \sigma) = \int_0^C \frac{c^{1-\sigma}}{\Omega(C, C_x; \tau, \sigma)} \frac{dZ(c)}{Z(C)} + \frac{Z(C_x)}{Z(C)} \int_0^{C_x} \frac{(c\tau)^{1-\sigma}}{\Omega(C, C_x; \tau, \sigma)} \frac{dZ(c)}{Z(C_x)} \tag{49}$$

Equation (47) will subsequently be used as the condition on optimal R&D investment. The right-hand side is the ex ante value of innovation, conditional on the implementability of the product. Following Haruyama and Zhao (2017), the left-hand side can be interpreted as an ex ante measure of fixed costs for developing a new quality vintage. In (48), it can be seen that all three kinds of fixed costs are included in the measure $\tilde{F}$, albeit with a different factor. The fixed costs of R&D are scaled by a factor $1/Z(C)$, which is greater than one. It states how often a firm has to make the investment into research before finding a product with sufficiently low marginal costs. Clearly, the lower the cut-off value C is, the more often, in expectation, a firm has to make this investment. Independent of the number of projects a firm has tried to implement, the fixed costs of production will only be paid once. Therefore, these costs enter linearly into the fixed cost measure. The last term in the measure, concerning the fixed costs of exporting, are scaled by a factor of $Z(C_x)/Z(C)$. This factor will in most cases be smaller than one, given that the cut-off value of exporting is presumably lower than the cut-off value for production.[15] As we are in the ex ante case, this scaling is quite intuitive, because not every firm is also going to be an exporter. So, the fixed costs of exporting should only enter in a reduced manner. In equation (49), it can be seen that $\Lambda(C, C_x; \tau, \sigma)$ is decreasing in the marginal cost c. As already discussed, higher marginal costs decrease profits by reducing the demand for the product. This effect in captured by the function. The first part of the function is for the domestic demand and the second part, including the trade costs, is for the foreign demand.[16]

The last equilibrium condition concerns the labour market. Combining the manufacturing labour (44) with labour demand for fixed costs and (9), the labour market can be characterised by:

$$L = \frac{R}{f_r} Z(C) \tilde{F}(C, C_x) + \frac{\sigma - 1}{\sigma} E(t) L \; \Theta(C, C_x; \tau, \sigma) \tag{50}$$

With that, we have established four equations to solve for the equilibrium.

Combining the two equations of (46), we can express the relationship between the cut-off value of domestic production and the one for exporting,

$$C = \left(\frac{f}{f_x} \right)^{\frac{1}{1-\sigma}} \tau \; C_x \tag{51}$$

This equation determines all combinations of cut-off values at which the two equations is (46) hold simultaneously. In case of $f < f_x$ and/or $\tau > 1$, the cut-off value for domestic production is above the cut-off value for exporting. That is, there is **selection into exporting**. An increase in the trade costs makes the domestic cut-off value larger in comparison to the exporting cut-off value. The intuition behind it is that, ex ante, the expected present

[15] At the most extreme level of trade liberalisation, the two cut-off values are identical, implying that the factor in front of the fixed costs of exporting is equal to unity.

[16] In Haruyama and Zhao (2017), a similar equation with a similar result is established. However, the cause is a different one. There, due to the Cobb-Douglas preferences, higher marginal costs reduce the profit margin, instead of reducing the demand as in the case of CES preferences. Nevertheless, the effect is the same.

discounted value of profits should be equal to the fixed costs of R&D. Now, if the trade costs increase, potential profits from exporting decrease. So, to keep the discounted value of profits constant, the profits from domestic production must increase, and this happens by an increase in the cut-off value of domestic production. Ex ante, when there still is uncertainty about the realised value of marginal costs, this is sufficient to keep the expected discounted value of profits constant. Alternatively, (51) can be reformulated to express C_x is term of C. Substituting the first equation of (46) into (47), and using the result from (51), it can be established that

$$f_r = f \int_0^C \left(\left(\frac{c}{C} \right)^{1-\sigma} - 1 \right) dZ(c) + \int_0^{C_x(C)} \left(\left(\frac{c}{C} \right)^{1-\sigma} \tau^{1-\sigma} f - f_x \right) dZ(c) \qquad (52)$$

which uniquely determines the equilibrium cut-off value for domestic production C. The right-hand side of the equation is the ex ante value of innovation, which must be equal to the fixed costs of research. It is rising in the cut-off value C, as it becomes more likely to succeed in research when the cut-off value is higher. The main determinant of the value of innovation is what can be called the distance to the cut-off, meaning the difference between the realised value of marginal cost and the cut-off value, at which the value of innovation is zero. In Melitz (2003), a similar equation relating firm productivity to the cut-off level of productivity can be established, where profits increase with distance to the cut-off level.

Similarly, the cut-off level for exporting can be found by combining the second equation of (46), (47) and (51):

$$f_r = \int_0^{C(C_x)} \left(\left(\frac{c}{C_x} \right)^{1-\sigma} \tau^{\sigma-1} f_x - f \right) dZ(c) + f_x \int_0^{C_x} \left(\left(\frac{c}{C_x} \right)^{1-\sigma} - 1 \right) dZ(c) \qquad (53)$$

This equation works in a similar fashion as the previous one. The right-hand side represents the ex ante value of innovation. The value is increase in the cut-off value for exporting C_x, because it is likelier to become an exporter when the cut-off value is higher, thereby increasing the potential value of the innovation.

In equilibrium, the shares of the different types of industries should be constant. Therefore, equation (40) can be set equal to zero to find the share of exporting industries in equilibrium.

$$N_A = \frac{Z(C_x)}{Z(C)} \qquad\qquad N_B = \frac{Z(C) - Z(C_x)}{Z(C)} \qquad (54)$$

Given that C and C_x are constant in equilibrium, so will be the share of exporting industries N_A.

The most important equation in this model is concerned with the number of researchers. Combining (47) and (50) yields:

$$R = \frac{f_r}{Z(C)\tilde{F}(C,C_x)} \left(\frac{\Lambda(C,C_x)L - (\sigma-1)\rho\ \Theta(C,C_x)\tilde{F}(C,C_x)}{\Lambda(C,C_x) + 2(\sigma-1)\ \Theta(C,C_x)} \right) \tag{55}$$

As in the standard Schumpeterian model, there is a scale effect present in this equation. The number of researchers is decreasing in the measure of fixed costs $\tilde{F}(C,C_x)$. The intuition behind it is obvious. Higher fixed costs mean higher upfront investment before any potential profits can be gained, making the investments less attractive in the first place. The function $\Lambda(C,C_x)$, capturing the demand effect of the marginal costs, has an ambiguous impact of research, depending on the size of the population. However, even for relatively small sizes, the positive effect outweighs the negative effect. The number of researchers is decreasing in the function $\Theta(C,C_x)$, which also is intuitive. The larger Θ is, the more people are employed in manufacturing, due to the lower prices of the competitively produced products. Given that more people work in manufacturing, there are fewer people available to work in research, so the number of researchers must decline. The parameters σ and ρ have the same direct effect on research as in the standard Model. A higher ρ implies that people are more impatient, therefore, future profits are less valuable to them. Clearly, when the gains from an investment are smaller, then the investment itself becomes less attractive. The effect of σ comes through the profits. The higher the elasticity of substitution, the fiercer the competition from the other industries. This increased inter-industry competition forces firms to reduce the price, which decreases the amount of profits generated from a product. So, a higher σ decreases the number of researchers because profits are lower.

Finally, the level of expenditure can be found by plugging (55) into (50):

$$E = \frac{2\sigma}{\Lambda(C,C_x) + 2(\sigma-1)\Theta(C,C_x)} \left(1 + \rho \frac{\tilde{F}(C,C_x)}{2L} \right) \tag{56}$$

The inside of the bracket is similar to the level of expenditure without firm heterogeneity. It is the sum of all income in one period. The first part is labour income and the second part is interest earned on equity investment, which is expressed in terms of fixed costs of production. In equilibrium, the interest rate is equal to ρ and every individual possesses an equal share of total equity in the world. The part in front of the bracket is some scaling.

4 Trade Liberalisation

4.1 Pareto Distribution

So far, no specific distribution function has be assumed. In order to facilitate the analysis of trade liberalisation in this section, a Pareto distribution function is assumed. This kind of distribution has been used in other trade literature, for example in Gustafsson and Segerstrom (2010), Helpman et al. (2008) or in Melitz and Ottaviano (2008). It appears to be a good representation of empirically observed firm heterogeneity (Del Gatto et al., 2006). Specifically, the cumulative distribution function takes the form

$$Z(c) = \left(\frac{c}{c_H}\right)^{\kappa}, \quad c \in (0, c_H)$$

where κ is the shape parameter and c_H is the scale parameter of the function. The probability density function is given by:

$$z(c) = \kappa \left(\frac{c}{c_H}\right)^{\kappa} c^{-1}$$

4.2 Autarky vs. Free Trade

The derivation of the autarky equilibrium is confined to appendix B. Free trade, in this context here, means that trade costs are absent and fixed costs of exporting are identical to the fixed costs of production in the domestic market. This implies $\tau = 1$ and $f_x = f$. From (51), it becomes immediately clear that $C = C_x$, the two cut-off values are identical. Therefore, there is no more selection into exporting under free trade, there is only selection into production itself. Consequently, the share of exporting industries, N_A, equals to one. That is, every industry is an exporting industry, there are no more industries where the highest quality good is not available in both countries. This will greatly simplify the equations describing the equilibrium. First, the equation on the cut-off value C is:

$$f_r = 2f \int_0^C \left(\left(\frac{c}{C}\right)^{1-\sigma} - 1\right) dZ(c)$$

It is evident that the profits from the domestic market are now identical to the profits from the foreign market. Under autarky, only the domestic market is available to generate profits, while under free trade the domestic and the foreign market are available. Therefore, the domestic market has to deliver higher profits under autarky than in the case of free trade, given that the total amount of profits must be equal to f_r in both cases. As already discussed, profits increase with distance to the cut-off value. In order to achieve higher profits in the domestic market, the cut-off value must therefore be higher under autarky than under free trade. Thus, it is more difficult to invent a new quality vintage with sufficiently low marginal costs in case of free trade. Applying the Pareto distribution function gives:

$$C^A = \left[\left(\frac{\kappa - \sigma + 1}{\sigma - 1}\frac{f_r}{f}c_H^\kappa\right)\right]^{\frac{1}{\kappa}} \tag{57}$$

$$C^{FT} = \left[\left(\frac{\kappa - \sigma + 1}{2(\sigma - 1)}\frac{f_r}{f}c_H^\kappa\right)\right]^{\frac{1}{\kappa}} \tag{58}$$

where the superscripts "A" and "FT" stand for autarky and free trade, respectively. Clearly, the cut-off value is smaller in the case of free trade, confirming the argumentation above.
Turning to the measure of fixed costs, we have:

$$F(C^A) = \frac{\kappa}{\kappa - \sigma + 1}f \tag{59}$$

$$F(C^{FT}) = \frac{2\kappa}{\kappa - \sigma + 1}f \tag{60}$$

The fixed cost measure $F(C)$ is exactly double in the case of free trade in comparison to autarky. In part, this is of course due to the fact that the fixed costs of production, f, have to be paid twice. The rest stems from the fact that entry into the market has become more difficult under free trade. The reduction in the cut-off value means that firms have to invest more often into research before they find a sufficiently profitable new quality vintage. So, the costs of research have gone up as well.
From this, we can calculate the number of researchers in both cases.

$$R^A = R^{FT} \equiv \frac{\sigma - 1}{\kappa\sigma}\left(L - (\sigma - 1)\rho\frac{\kappa f}{\kappa - \sigma + 1}\right) \tag{61}$$

Surprisingly, and in stark contrast to the central result of Haruyama and Zhao (2017), the number of researchers under free trade is identical to the number of researchers under autarky. However, a final conclusion on this result should await the analysis of partial trade liberalisation, where $\tau > 1$ and $f_x > f$ still holds.
Finally, the welfare effects of free trade can be calculated. Welfare depends on three things, on expenditure, the price index and the growth rate (see equation (31)). The level of expenditure E is the same in both cases. Although there are more assets in the case of free trade, they have to be shared among more people, which leaves the amount of assets per head constant. Similarly, the rate of technological progress stays the same. In the case of trade with firm heterogeneity, the growth rate can be expressed as:

$$g = (\lambda - 1)Z(C)(2I)$$

The fact that, now, two countries contribute to research increases the growth rate. Simultaneously, however, the probability of discovering a sufficiently profitable new product has decreased given that the cut-off value is lower. In sum, these two effects cancel out and the growth rate under free trade is identical to the growth rate under autarky. Any changes in welfare are thus solely determined by the price index. See (65) in appendix B for the exact formulation. The price index depends on the average marginal costs of production, which in turn depends on the cut-off value C. The lower C, the lower the average marginal costs and thus the lower the price index. In case of free trade, the cut-off value was found to be lower than in the case of autarky, implying that the price index is lower under free trade. Therefore, welfare under free trade is strictly larger than under autarky, owing to an increase in purchasing power through lower prices.

4.3 Partial Liberalisation

Partial trade liberalisation, or just trade liberalisation, can happen in two ways. Either a reduction in the trade costs τ or the fixed costs of exporting f_x. In order to make the analysis of the equations more understandable, this part will be done numerically. For most of the parameters in this model, an exact number is much less important than the relation to each other. Only for the parameters σ and κ, which can both be micro-founded, is it important to choose a realistic value. For the Pareto shape parameter κ, a value of 2 seems to be realistic and has been used in corresponding literature (Del Gatto et al., 2006;

Gustafsson and Segerstrom, 2010; Melitz and Ottaviano, 2008). Concerning the elasticity of substitution σ, there are several approaches to estimate it. One way would be to estimate it using bilateral trade flow models. This approach leads to estimates ranging from 5 up to 10.[17] Another approach would be to deduce it from firm markups. Recent estimates conclude that firm markups have risen substantially over the last years and may well reach 40-50 per cent (De Loecker et al., 2020; Edmond et al., 2018). In the present framework, such estimates would imply an elasticity of substitution of 3 to 4, which is definitively lower than the estimates from the trade flow approach. Equation (62) shows the explicit solutions to the cut-off conditions with Pareto distribution. As can be seen, any meaningful solution to this requires that $\sigma < \kappa + 1$. In order to reconcile this condition with the empirical evidence on both parameters, I choose to set $\kappa = 2.5$ and $\sigma = 3.3$, which is a reasonable value according to the evidence on firm markups. Djankov et al. (2002) present evidence on the costs of setting up a start-up firm. These costs are quite considerable in most countries. What is more difficult is to differentiate the costs for domestic and foreign firms. It is highly plausible that foreign firms face higher costs than domestic firms, for various reasons. Some of them might be of regulatory nature, for example more administrative paperwork or higher fees. Non-regulatory reasons include language barriers or unfamiliar market and regulatory environments. Of course, a firm from a high-regulation country might actually find a more friendly environment in a foreign country. However, in most cases, it should hold that fixed costs of exporting are higher than the fixed costs of domestic production. Therefore, the assumption that $f_x > f$ is reasonable. I set f equal to 10 and f_x in a range of 10-16, ensuring that f_x is never smaller than f. Depending on industry, research expenditure can be many times smaller or larger than the fixed costs of production. Given, however, that the parameter f_r cannot fundamentally alter the subsequent results, the exact value is not of much importance. I choose to set $f_r = 20$, implying that research costs are double the costs of setting up production. Table 1 provides an overview over all parameter values.

$$
\begin{aligned}
C &= \left[\frac{\kappa-\sigma+1}{\sigma-1} c_H^{\kappa} \frac{f_r}{f} \left(\frac{1}{1+\left(\frac{f_x}{f}\right)^{\frac{\kappa-\sigma+1}{1-\sigma}} \tau^{-\kappa}}\right)\right]^{\frac{1}{\kappa}} \\
C_x &= \left[\frac{\kappa-\sigma+1}{\sigma-1} c_H^{\kappa} \frac{f_r}{f_x} \left(\frac{1}{1+\left(\frac{f}{f_x}\right)^{\frac{\kappa-\sigma+1}{1-\sigma}} \tau^{\kappa}}\right)\right]^{\frac{1}{\kappa}}
\end{aligned}
\tag{62}
$$

[17] See Anderson and van Wincoop (2004) for an overview of the literature.

σ	3.3
κ	2.5
c_H	20
τ	1-5
f	10
f_r	20
f_x	10-16
c_L	3
ρ	0.04
L	10'000

Table 1: Numerical values

4.3.1 Manufacturing Productivity

The effects of trade liberalisation in either τ or f_r on the cut-off values C and C_x can be seen in figure 2. The cases of trade liberalisation are considered in isolation. That is, if there is trade liberalisation in τ, then f_r stays constant, and vice versa. In either case, some intermediary value is chosen for the constant. The result can be summarised as follows:

Proposition 2. *Trade liberalisation in τ or f_x reduces C and increases C_x*

Proposition 2 implies that entry into production becomes more difficult with trade liberalisation, making successful innovations less likely. At the same time, exporting becomes easier and more likely, once a firm has a successful innovation. Define average manufacturing productivity as:

$$\int_0^C c^{-1} \quad \frac{dZ(c)}{Z(C)}$$

By reducing the cut-off value for production, trade liberalisation clearly increases average manufacturing productivity, making the economy more efficient at producing its goods. This efficiency gain is purely static, there is no change in the growth rate. Inefficient firms are driven out of the market, while more efficient firms can expand their market share. This result has already been found by Melitz (2003), where trade liberalisation causes a static, one-time increase in manufacturing productivity. It directly follows from proposition 2 is that $Z(C)$ decreases and $Z(C_x)$ increases (see figure 3). This means that the share of exporting industries (and firms) increases. A larger part of the economy is now also exporting. Conversely, this also implies that the share of industries closed to trade has been reduced, more industries opened up to trade owing to the reduced costs attached to trade. Thus, there is a reallocation of resources from non-exporting industries to exporting industries. These results are in line with Haruyama and Zhao (2017) and many empirical results (see for example Bernard et al. (2006); Tybout (2001)).

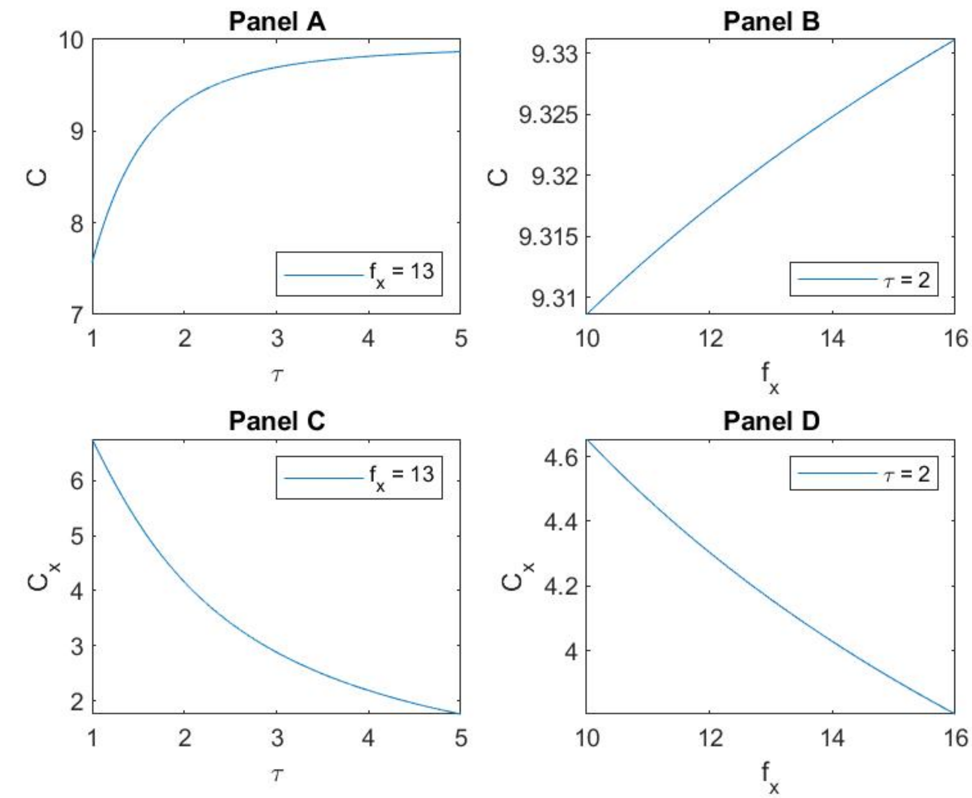

Figure 2: The effect of trade liberalisation on C and C_x

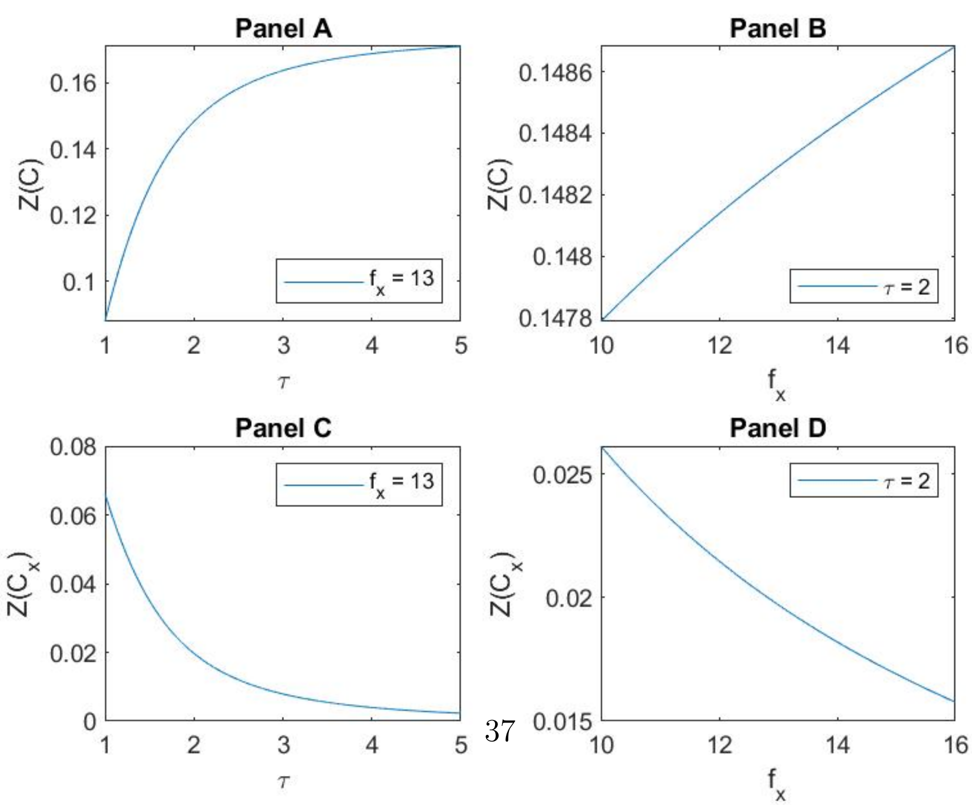

Figure 3: The effect of trade liberalisation on Z(C) and $Z(C_x)$

4.3.2 R&D Investment

In order to analysis the effect of trade liberalisation on the number of researchers, we still need the function $\tilde{F}(C, C_x)$, $\Theta(C, C_x)$ and $\Lambda(C, C_x)$. Start with the measure of fixed costs. As can already be seen from the autarky vs. free trade analysis, the measure of fixed costs increases when moving out of autarky. Indeed, figure 4 confirms this result. Trade liberalisation in either τ or f_x increases the fixed costs. Two factors contribute to this increase. First, a firm must, in expectation, invest more into research before finding a sufficiently profitable innovation, because the cut-off value has decreased. Second, given that more industries are open to trade, it is more likely that the firm will also be exporter. Thus, it becomes more likely that it has to pay the fixed costs of exporting, further increasing the total measure of fixed costs. It has already been established that $\tilde{F}(C, C_x)$ has a negative impact on the number of researchers.

The effects of trade liberalisation on $\Theta(C, C_x)$, the function from the manufacturing labour demand, are represented in figure 5. Remember than an increase in $\Theta(C, C_x)$ increases the number of workers employed in manufacturing, *ceteris paribus* leaving fewer people to work in research. Looking at equation (63), it can be seen that the difference between the numerator and the denominator stems from the share of industries in which the second-highest quality vintage is produced at competitive prices. Continued trade liberalisation shrinks the share of these industries, continuously closing the discrepancy. So, trade liberalisation decreases $\Theta(C, C_x)$, by increasing the share of industries in which CES pricing occurs. This reduction in $\Theta(C, C_x)$ will have a positive impact on the number of researchers.

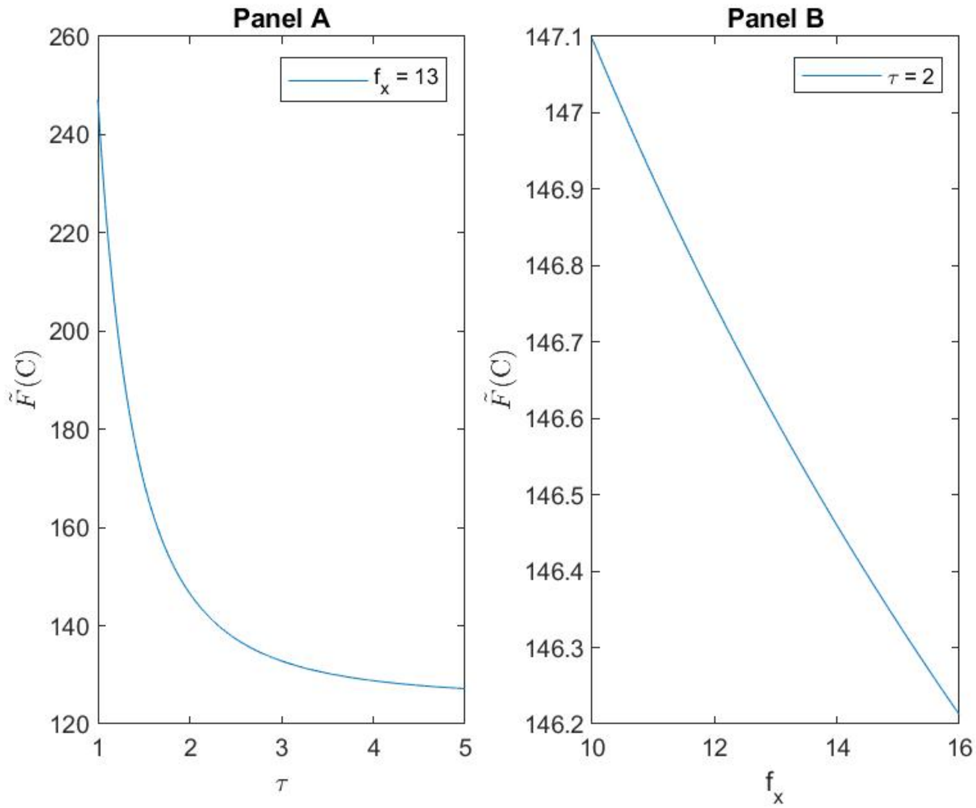

Figure 4: The effect of trade liberalisation on $\tilde{F}(C, C_x)$

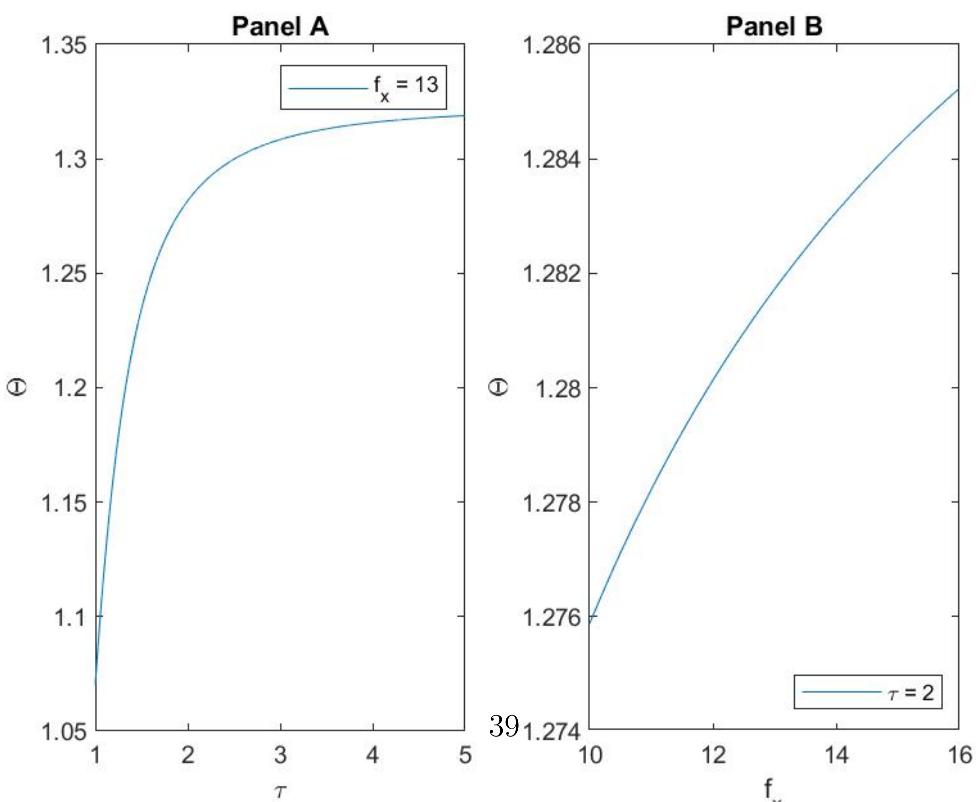

Figure 5: The effect of trade liberalisation on $\Theta(C, C_x)$

Last, there is $\Lambda(C, C_x)$. It captures the demand effect of the marginal costs (see (49)). Trade liberalisation, both in τ and f_x, has a positive effect on $\Lambda(C, C_x)$, as can be seen in figure 6. Although, due to the assumption of CES preferences, the profit margin is constant, trade liberalisation nevertheless has a positive effect on profits. The reduction in trade costs is passed on to the consumers, thereby reducing prices of imported goods, which in turn increases demand for these products. The increased demand allows firms to enjoy higher profits, even though the profit margin is unaffected. Because, at the same time, trade liberalisation makes firms more productive, this increase in demand does not require additional labour in manufacturing. In fact, it even allows to free up some workers for research. So, $\Lambda(C, C_x)$ does have a positive impact on the number of researchers, because it makes successful innovations more profitable. In Haruyama and Zhao (2017), there is a similar effect operating, although in their model, the cause is a different one. Due to the Cobb-Douglas preferences in their model, trade liberalisation increase profits margins for firms, while demand for the products remains constant. So, the firms keep the gains from trade liberalisation for themselves instead of passing them on to the consumers. However, the effect is the same, firm profits increase due to trade liberalisation.

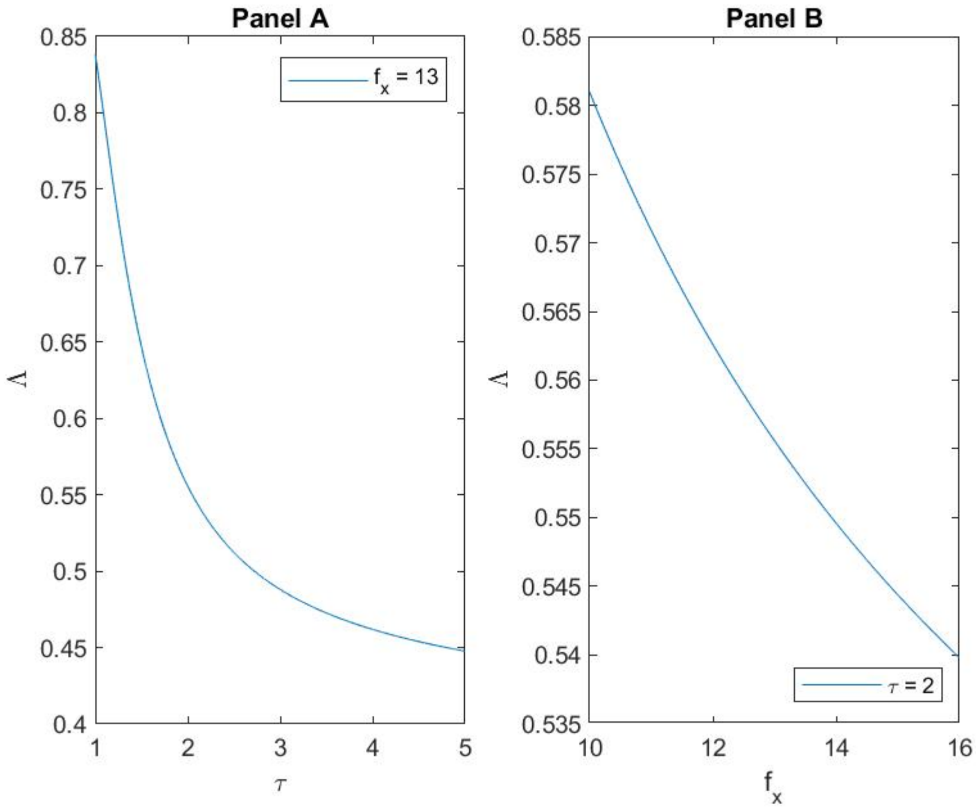

Figure 6: The effect of trade liberalisation on $\Lambda(C, C_x)$

Finally, the number of researchers can be constructed using all the functions just described (see again (55) for the formulation). The behaviour of the functions $Z(C)$, $\Theta(C, C_x)$ and $\Lambda(C, C_x)$ all encourage research investment, while $\tilde{F}(C, C_x)$ discourages research investment. The overall effect can be seen in figure 7. Clearly, the number of researchers increases during the course of trade liberalisation. The central result of Haruyama and Zhao (2017), that trade liberalisation increases the incentives for innovation, can be confirmed in this CES preferences framework. However, unlike in their model, this is no general proof of this result, but rather dependent on the choice of parameters. At higher values of σ and κ, this positive relationship between trade liberalisation and research incentives breaks down and eventually turns negative. Trade liberalisation would then actually reduce the number of researchers. In (55), it can be seen that σ has a direct, negative effect on R, by increasing the weight of $\tilde{F}(C, C_x)$. Additionally, there are several indirect effects present as well, which accentuate with higher values of σ. So, it would be accurate to state that the result of Haruyama and Zhao (2017) also holds in the CES framework, provided that the elasticity of substitution is not too high. The values of σ, for which the result turns upside down, start between 7 or 8. This would be perfectly in line with the estimations from the trade literature. However, it would also imply implausibly small profit margins and implausibly high values of κ, given that $\sigma < \kappa + 1$ has to hold for any reasonable solution. So, from the point of view of firm markups, the required level of σ for a change in the results are too high to be realistic. This suggests that trade liberalisation does have a positive effect on the number of researchers for plausible parameter values. Thus, we can establish the following proposition:

Proposition 3. *Trade liberalisation in τ or f_x increases the number of researchers for small values of σ.*

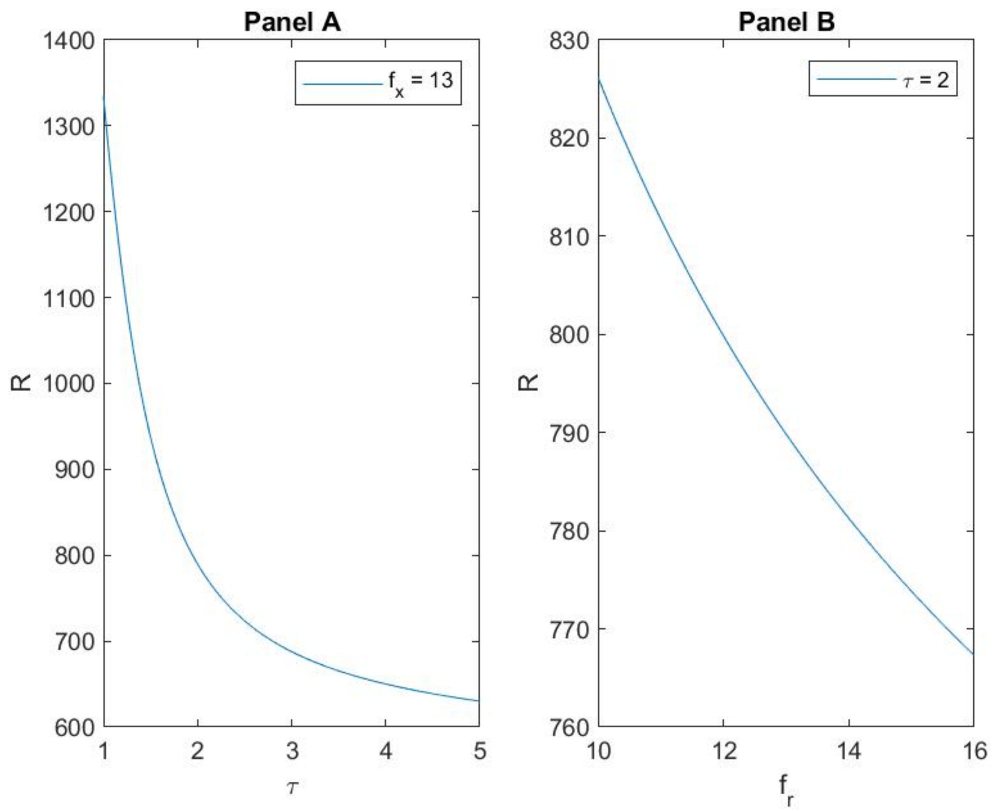

Figure 7: The effect of trade liberalisation on R

In the previous subsection, it has been shown that the number of researchers in autarky and free trade are identical. Combining this fact with the result presented here, it follows that autarky and free trade are the maxima of the function. Therefore, moving out of autarky would lead to a massive drop in the number of researchers. During the course of trade liberalisation, this number recovers again until, finally, it is back at its old level. This appears to be an oddity of the model set-up, given that with firm homogeneity, the move out of autarky behaves quite differently (see again proposition 1), and that it makes intuitively little sense why there should be such a massive drop in research investment. Unfortunately, Haruyama and Zhao (2017) do not calculate the autarky equilibrium. So, it is not known whether their model exhibits a similar behaviour.[18]

5 Conclusion

The framework presented in this study can confirm many results of other studies on firm heterogeneity. Trade liberalisation increases the average manufacturing productivity, making the economy more efficient. It does so by driving less efficient firms out of the market, while only allowing more efficient firms to enter. There is a reallocation of resources from non-exporting sectors to exporting sectors of the economy. At the same time, the exporting sector expands, more and more firms start exporting when trade costs are reduced.

This study has shown that the central result of Haruyama and Zhao (2017), that trade liberalisation increases the number of researchers, and thus the incentives for innovation, also holds true in a CES framework, as long as the elasticity of substitution is reasonably small. At higher values, the whole result turns upside down and trade liberalisation actually becomes harmful for the innovation incentives. Nevertheless, I am able to show the results from Haruyama & Zhao are not caused by their assumption of a Cobb-Douglas production function. Rather, the difference to other studies, which were not able to produce such a positive link between trade liberalisation and growth, should therefore stem from the usage of a quality-ladder model of growth instead of the commonly used variety-expansion approach.

The model of firm homogeneity in the first section has shown that, although the move out of autarky is clearly growth promoting, further trade liberalisation has no effect on growth. This is due to the fact that the number of researchers is independent of the trade costs. Under firm heterogeneity, however, the number of researchers is affected by changes in the trade costs. An intuitive explanation for this would be that certain firms can expand their market share in the case of trade liberalisation at the cost of less efficient firms, which lose

[18]This model behaviour could be reconciled with basic intuition if the definition of autarky is altered. If autarky is redefined as "trade with prohibitively high trade costs", then the sudden drop when moving out of autarky disappears, because this move would then be identical to a move along the curve in figure 7.

some market share or are driven out of the market entirely. So, having comparatively small marginal costs of production becomes more attractive, meaning, more profitable. Therefore, new entrants are willing to invest more resources into research, such that they can profit from this situation as well.

This study has limited itself to provide a theoretical framework to analyse trade liberalisation in the context of quality-enhancing innovation. A proposal for future research would be to study the exact conditions under which trade liberalisation starts to exhibit a negative effect on the incentives for innovation and explain the economic rationale behind this complete turnover.

Appendix A

A.1 Price Index

The definition of the price index is still given by (6), namely:

$$P(t) = \left[\int_0^1 q(n,i,t)\ p(n,i,t)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

The price charged in an industry depends on the category of the industry (type-A or type-B, see section on industry dynamics). Firms in the type-B industries are non-exporting firms, either because they produce the highest quality good at too high marginal cost or they produce the second-highest quality good because the quality leader from abroad has too high marginal costs. Due to the assumption that the trading partners are structurally identical, these non-exporting quality leaders are evenly distributed across both countries. So, in one half of all type-B industries, the CES pricing applies. In the other half, the competitive price c_L is applied. Turning to the type-A industries, where the product is also exported, half of these are located in the home country and half are located in the foreign country, again due to the symmetry of the countries. In the type-A industries, only the CES price applies. However, for those products which are imported, the trade costs τ have to the added to the price. In mathematical terms, this can expressed as[19]:

$$P(t) = \left[\int_0^{N_B/2} q(i)\ c_L^{1-\sigma}\ di + \int_0^{N_B/2} q(i)\ \frac{\sigma}{\sigma-1} \int_{C_x}^{C} c^{1-\sigma} \frac{dZ(c)}{Z(C)-Z(C_x)}\ di \right.$$
$$\left. + \int_0^{N_A/2} q(i)\ \frac{\sigma}{\sigma-1} \int_0^{C_x} (c\tau)^{1-\sigma} \frac{dZ(c)}{Z(C_x)}\ di + \int_0^{N_A/2} q(i)\ \frac{\sigma}{\sigma-1} \int_0^{C_x} c^{1-\sigma} \frac{dZ(c)}{Z(C_x)}\ di \right]^{\frac{1}{1-\sigma}}$$

$$P(t) = Q^{\frac{1}{1-\sigma}} \left[\frac{N_A}{2} \frac{\sigma}{\sigma-1} \int_0^{C_x} (c\tau)^{1-\sigma} \frac{dZ(c)}{Z(C_x)} + \frac{N_A}{2} \frac{\sigma}{\sigma-1} \int_0^{C_x} c^{1-\sigma} \frac{dZ(c)}{Z(C_x)} \right.$$
$$\left. + \frac{N_B}{2} c_L^{1-\sigma} + \frac{N_B}{2} \frac{\sigma}{\sigma-1} \int_{C_x}^{C} c^{1-\sigma} \frac{dZ(c)}{Z(C)-Z(C_x)} \right]^{\frac{1}{1-\sigma}}$$

[19]In order to factor out the average quality level Q, it is implicitly assumed that average quality does not differ across industry types

Finally, this can be reduced to:

$$P^{1-\sigma} = Q\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\Bigg[\frac{N_A}{2}(1+\tau^{1-\sigma})\int_0^{C_x} c^{1-\sigma}\frac{dZ(c)}{Z(C_x)} + \frac{N_B}{2}\left(c_L^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} + \int_{C_x}^{C} c^{1-\sigma}\frac{dZ(c)}{Z(C)-Z(C_x)}\right)\Bigg] \quad \text{(A.1)}$$

A.2 Manufacturing Labour Demand

The demand for manufacturing labour is given by (see (13)):

$$l_m = \int_0^1 c \cdot y(i)\ di$$

where $y(i)$ is given by (5) and c is the marginal cost of a specific firm. As with the price index, we have to keep track of four different cases. Namely, first, the highest quality good is produced at home, but is not exported. Second, the highest quality good is produced abroad, but not exported, therefore only the second-highest quality is available. Third, the highest quality good is produced at home and also exported. Forth, the highest quality good is produced abroad and also exported. In the first case, manufacturing labour demand is given by:

$$l_{B1} = \int_0^{N_B/2} \frac{q(i)EL}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_{C_x}^{C} c^{1-\sigma}\frac{dZ(c)}{Z(C)-Z(C_x)}\ di$$

Similarly, the labour demand for the other three cases can be expressed by:

$$\begin{aligned}
l_{B2} &= \int_0^{N_B/2} \frac{q(i)EL}{P^{1-\sigma}} c_L^{1-\sigma}\ di \\
l_{A1} &= \int_0^{N_A/2} \frac{q(i)EL}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_0^{C_x} c^{1-\sigma}\frac{dZ(c)}{Z(C_x)}\ di \\
l_{A2} &= \int_0^{N_A/2} \frac{q(i)EL}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_0^{C_x} (c\tau)^{1-\sigma}\frac{dZ(c)}{Z(C_x)}\ di
\end{aligned}$$

Adding up the four parts, and assuming the average quality level across industry types does not differ, we can write:

$$l = \frac{QEL}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\Bigg[\frac{N_B}{2}\int_{C_X}^{C} c^{1-\sigma}\frac{dZ(c)}{Z(C)-Z(C_X)} + \frac{N_B}{2}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma} c_L^{1-\sigma}$$
$$+ \frac{N_A}{2}\int_0^{C_X} c^{1-\sigma}\frac{dZ(c)}{Z(C_X)} + \frac{N_A}{2}\int_0^{C_X}(c\tau)^{1-\sigma}\frac{dZ(c)}{Z(C_X)}\Bigg]$$

Using the solution for the price index (A.1), this can be simplified to:

$$l_m = \frac{\sigma-1}{\sigma}E(t)L\ \Theta(C, C_X; \tau, \sigma) \tag{A.2}$$

where

$$\Theta(C, C_x; \tau, \sigma) = \frac{\Xi(C, C_x; \tau, \sigma)}{\Omega(C, C_x; \tau, \sigma)} = \frac{\frac{N_B}{2}\left(c_L^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma} + \int_{C_x}^{C} c^{1-\sigma}\frac{dZ(c)}{Z(C)-Z(C_x)}\right) + \frac{N_A}{2}\left(1+\tau^{1-\sigma}\right)\int_0^{C_x} c^{1-\sigma}\frac{dZ(c)}{Z(C_x)}}{\frac{N_B}{2}\left(c_L^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} + \int_{C_x}^{C} c^{1-\sigma}\frac{dZ(c)}{Z(C)-Z(C_x)}\right) + \frac{N_A}{2}\left(1+\tau^{1-\sigma}\right)\int_0^{C_x} c^{1-\sigma}\frac{dZ(c)}{Z(C_x)}} \tag{63}$$

Equation (A.2) is identical to (44).

In the section without firm heterogeneity, manufacturing labour demand was given by $l = \frac{\sigma-1}{\sigma}EL$, implying that Θ was equal to one. With firm heterogeneity, Θ is larger than one (which becomes obvious when analysing the first term of the nominator and denominator, the rest is the same). Thus, manufacturing labour demand is larger in the case of firm heterogeneity, ceteris paribus. This is because a fraction of all products, exactly $N_B/2$, is produced at competitive prices, thereby increasing demand. So, the increase in labour demand is driven by larger product demand due to lower prices in certain industries.

Appendix B: Autarky Equilibrium

Most of the equations for the autarky equilibrium can easily be derived from the ones for the trade equilibrium. It is important to keep in mind that the replacement rate is again only $I(t)$, and not $2I(t)$ as in the case of trade. Using the formula for the manufacturing labour, the reader can easily verify that it can be expressed as:

$$l_m = \frac{\sigma - 1}{\sigma} E(t) L \tag{64}$$

which is in fact the same as in the case of firm homogeneity. The price index, which has also been used in the equation above, is given by:

$$P^{1-\sigma} = Q \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \int_0^C c^{1-\sigma} \, \frac{dZ(c)}{Z(C)} \tag{65}$$

With this information, the three conditions for the unknowns C, E and R can easily be derived from those in the main text. The autarky equilibrium is then given by:

$$f_r = f \int_0^C \left[\left(\frac{c}{C} \right)^{1-\sigma} - 1 \right] dZ(c) \tag{66}$$

$$R = \frac{f_r}{\sigma Z(C) F(C)} \Big(L - (\sigma - 1) \rho F(C) \Big) \tag{67}$$

$$E = 1 + \rho \frac{F(C)}{L} \tag{68}$$

where

$$F(C) = \frac{f_r}{Z(C)} + f \tag{69}$$

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